MODELING OF HUMAN MOTOR CONTROL AND ITS APPLICATION IN HUMAN INTERACTION WITH MACHINES

by

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Human civilization started with the invention of tools which enhanced and expanded human motor capability. With the recent development of virtual reality technology and artificial intelligence, the interaction between humans and machines has become more and more intricate. A better understanding of our motor system and the way it interacts with machines will allow us to better design intelligent devices. However, previous works in motor control modeling mostly focused on linear dynamics and had limitations in incorporating the process of learning.

A musculoskeletal model based on mechanical principles and a motor control model based on Bayesian probability are proposed in this study. The probability-theoretical formulation of the problem not only facilitates the understanding of motor learning but also transforms nonlinear dynamics into linear problems. Using these models, the interactions in which both human and machine are capable of learning and adapting are formulated and analyzed. Intelligent control policies for machine imitating the human motor control are proposed. Simulation results are also presented.

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1.0 INTRODUCTION

Man the Tool-maker

Kenneth Page Oakley, 1949 [13]

Toolmaking is a characteristic once used by anthropologists [21] to distinguish humans from animals. Although many animals are also capable of creating their own tools, humans' capability to extend our ability beyond the biology limitation lies in the core of civilization.

The tools humans have been using have evolved dramatically. They started with slightly modified branches and stones with very limited usage. Then we invented advanced and sophisticated mechanics. The industrial revolution brought us autonomous machines. The information era has arrived with devices of such great capability that they even defeat human intelligence in many tasks.

Despite these achievements, the very human intelligence involved in tool-making is still little known. How do we use our human body to control the tools? How does the sophistication of the tools influence our motor system? What will be the further amelioration of the tool-making? It's not until the recent development of neuroscience, control theory, computer and information science, that these fundamental questions started to be addressed.

Woodworth's Ph. D. dissertation in 1899 [22] is one of the first works on the production of voluntary movement. He conducted extensive experiments on the accuracy of human movement and the different factors contributing to its variability and proposed one of the first models of motor control, which can be summarized as a combination of feedforward and feedback using the language of the modern control theory [6]. In the mid-20th century, Bernstein suggested a multi-level hierarchy in movement construction and related it to different structures in the nervous system. He also pointed out the problem of motor redundancy, showing the importance of the control theory in movement study.

Recent models of motor control include the observer-predictor-feedback model, the optimal feedback control model, the intermittent control model etc. However, these models are usually focused on linear dynamics and had limitations in incorporating the process of learning. These weaknesses will prevent us from studying human motor behavior in complex situations. This thesis gives a new control theoretical model of the human motor system and its control and uses it to study the interaction between humans and tools or, in their modern version, machines. The feature of co-learning, which emerges when the machine has the human-like intelligence, will also be discussed. The focus in this thesis is the Bayesian representation of our body and the exterior world in our brain during the processes of system identification, state estimation, and optimal control. The probabilistic formulation of the problem allows us to transform the nonlinear stochastic dynamics into a linear deterministic problem. The framework provided in this thesis will be extremely useful for both practical applications and theoretical studies.

Hopefully, the study of purposeful biological movement and human interaction with the machines will not only help us revealing the underlying secret of engineering and justify the hypothesis of technological singularity, but also promote the understanding of our relations with the physical world and with each other.

2.0 BACKGROUND

2.1 CONTROL THEORY

Control theory studies the way a dynamical system can be controlled through a certain control policy. Classical control methods for linear time-invariant (LTI) systems, notably the PID controller, have achieved huge success in the industry. But for controlling nonlinear dynamics, which are omnipresent in biology and in the physical world, advanced control methods developed from analytic mechanics are indispensable. Some basic definitions in modern control theory are given below.

The set of physical variables characterizing a dynamical system can be modeled by a vector called state $x \in X = \mathbb{R}^n$, such that the evolution of this system can be characterized by a function of time called trajectory $x_t : I \subseteq \mathbb{R} \to X$, satisfying an ordinary differential equation (ODE) $\forall t \in I, \dot{x}_t(t) = f(x_t(t), t)$, where the function f is called a vector field.

A physical variable $u \in U = \mathbb{R}^m$ is a control of the dynamical system if the evolution of the system depends on u, and the dynamics of the controlled system can be written as $\dot{x}_t(t) = f(x_t(t), u_t(t), t)$, where the function $u_t : I \to U$ is called a control policy. If there is a function $u_x : X \times I \to U$ such that $u_t(t) = u_x(x_t(t), t), \forall t \in I$, then we call it a feedback control.

For an introduction to control theory, see [1].

2.1.1 Optimal Control

An optimal control problem consists of designing the control of a system so that the system has an optimal behavior. Formulating the control problem as an optimization problem clarifies the control goal and allows rigorous mathematical treatment of the problem. A typical optimal control problem can be formulated as the following.

Given the vector field f and the initial state x_0 of a system, define for all x_t and u_t satisfying the Cauchy problem

$$\begin{cases} \dot{x}_t(t) = f(x_t(t), u_t(t), t)\mathrm{d}t, \forall t \in [0, T] \\ x_t(0) = x_0 \end{cases} \tag{2.1}$$

a functional called cost $S[x_t, u_t] = \int_0^T L(x_t(t), u_t(t), t)\mathrm{d}t$, where the function L is called the Lagrangian. The problem of finding a control policy u_t^* and a state trajectory x_t^*, such that $S[x_t^*, u_t^*]$ is the minimum of all the possible values of S, is called an optimal control problem. For the simplicity of notation, we also use the variable x to denote its on-shell value $x_t(t)$ when there is no confusion, and then we can write the optimal control problem as

$$\begin{aligned} &\min_{u,x} \int_0^T L(x, u, t)\mathrm{d}t \\ &\text{s.t. } \dot{x} = f(x, u, t). \end{aligned} \tag{2.2}$$

This is an optimization problem constrained by an ODE. There are two major approaches to solve this problem. The first approach is the Pontryagin's maximum principle (PMP), which states that if u_t^* and x_t^* is a solution to (2.2), then we have

$$H(x_t^*(t), u_t^*(t), p_t^*(t), t) \leq H(x_t^*(t), u, p_t^*(t), t), \forall u \in U, \forall t \in [0, T] \tag{2.3}$$

where the function $H : (x, u, p, t) \mapsto p^{\mathrm{T}} f(x, u, t) + L(x, u, t)$ is called the Hamiltonian in which the variable $p \in \mathbb{R}^n$ is called the costate, and the trajectory of the optimal costate p_t^* satisfies the Cauchy problem

$$\begin{cases} (\dot{p}_t^*)^{\mathrm{T}}(t) = -\partial_x H(x_t^*(t), u_t^*(t), p_t^*(t), t), \forall t \in [0, T] \\ p_t^*(T) = 0. \end{cases} \tag{2.4}$$

Thus by solving simultaneously the state equation $\dot{x} = \partial_p H$, the costate equation $\dot{p} = -\partial_x H$, and the stationary condition $\partial_u H = 0$, we can find the optimal control policy.

While the PMP is a necessary condition for an optimal control, another approach using the Hamilton-Jacobi-Bellman (HJB) equation is a necessary and sufficient condition. The HJB equation

$$\frac{\partial V}{\partial t}(x,t) + \min_{u_t} H\left(x, u_t(t), \frac{\partial V}{\partial x}(x,t), t\right) = 0 \tag{2.5}$$

is a partial differential equation (PDE) of the cost-to-go function V, defined as

$$\begin{aligned} V(x,t) = \min_{u_t, x_t} & \int_t^T L(x_t(s), u_t(s), s)\mathrm{d}s \\ & \text{s.t.}\, \dot{x}_t(s) = f(x_t(s), u_t(s), s), \forall s \in [0,T] \\ & x_t(T) = x. \end{aligned} \tag{2.6}$$

By minimizing H and solving the HJB, we obtain the optimal feedback control.

The linear quadratic regulation (LQR) problem is an important type of optimal control problem. In this problem, the Lagrangian L is quadratic in x and in u, the vector field f is linear in x and in u. So the LQR problem is written as

$$\begin{aligned} & \min_{u,x} \int_0^T (x^{\mathrm{T}} Q(t) x + u^{\mathrm{T}} R(t) u)\mathrm{d}t \\ & \text{s.t.}\, \dot{x} = A(t)x + B(t)u. \end{aligned} \tag{2.7}$$

Using the PMP or the HJB, we obtain a feedback control as its solution

$$u_x(x,t) = -R^{-1}(t)B^{\mathrm{T}}(t)P(t)x \tag{2.8}$$

where the time-dependent positive definite matrix $P(t)$ satisfies the Riccati equation

$$\dot{P} + Q + A^{\mathrm{T}}P + PA - PBR^{-1}B^{\mathrm{T}}P = 0. \tag{2.9}$$

For a detailed review of optimal control theory, see [8].

2.1.2 Stochastic Process

A dynamical system is a stochastic process if the state variable x is not only a function of time t but also a function of the outcome from a sample space. The simplest non-trivial continuous stochastic process is the Wiener process. It models the Brownian motion, and its time-derivative models the Gaussian white noise.

To model the evolution of a stochastic state variable x, an ODE is no longer sufficient. Instead, we need a stochastic differential equation (SDE) called the Langevin equation

$$\mathrm{d}x_t = \mu(x_t, t)\mathrm{d}t + \sigma(x_t, t)\mathrm{d}W_t \tag{2.10}$$

which has an additional term called diffusion attributed to some Wiener process W_t.

The same process can also be modeled by the Kolmogorov equation. It's a PDE of the probability density function (PDF) of the state at time t, i.e. $p(x,t) = \partial_x \mathbb{P}(x_t(t) \leq x)$. Given the initial condition $p(x, 0)$, the Kolmogorov equation has two parts

$$\begin{aligned} \partial_t p &= -\sum_i \frac{\partial(f_i p)}{\partial x_i} + \frac{1}{2}\sum_{ijk} \frac{\partial^2(\sigma_{ik}\sigma_{jk}p)}{\partial x_i \partial x_j}, \forall t \geq 0 \\ -\partial_t p &= \sum_i f_i \frac{\partial p}{\partial x_i} + \frac{1}{2}\sum_{ijk} \sigma_{ik}\sigma_{jk} \frac{\partial^2 p}{\partial x_i \partial x_j}, \forall t \leq 0. \end{aligned} \tag{2.11}$$

The part for $t \geq 0$ is called the Kolmogorov forward equation or the Fokker-Planck equation. The part for $t \leq 0$ is called the Kolmogorov backward equation. The time-irreversibility is due to the causal nature of the stochastic processes. To simplify the notation, define the Kolmogorov forward operator

$$\hat{L} : p \mapsto -\sum_i \frac{\partial(f_i p)}{\partial x_i} + \frac{1}{2}\sum_{ijk} \frac{\partial^2(\sigma_{ik}\sigma_{jk}p)}{\partial x_i \partial x_j} \tag{2.12}$$

and the Kolmogorov backward operator

$$\hat{L}^\dagger : p \mapsto \sum_i f_i \frac{\partial p}{\partial x_i} + \frac{1}{2}\sum_{ijk} \sigma_{ik}\sigma_{jk} \frac{\partial^2 p}{\partial x_i \partial x_j}. \tag{2.13}$$

2.1.3 Bayesian Inference

A fundamental theorem in the probability theory is the Bayes' rule, which states that, for all events A and B, and any partitioning $(A_i)_i$ of the sample space, we have

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\sum_i \mathbb{P}(B|A_i)\mathbb{P}(A_i)}. \tag{2.14}$$

This equation relates the prior knowledge $\mathbb{P}(A)$ of the event A and its posterior knowledge $\mathbb{P}(A|B)$ through an observed event B.

An application of the Bayes' theorem is the information filtering, i.e. extracting the information a measurement provides for an unknown parameter, same as the state estimation problem in control theory, as in the following example.

Given the following dynamics of the state x and the measurement y

$$\begin{cases} \mathrm{d}x_t = f(x_t)\mathrm{d}t + \sigma(x_t)\mathrm{d}\omega_t \\ \mathrm{d}y_t = h(x_t)\mathrm{d}t + \mathrm{d}\nu_t \end{cases} \tag{2.15}$$

where ω_t and ν_t are two independent Wiener processes. The Zakai equation is a stochastic partial differential equation of the non-normalized PDF $\rho(x,t)$ of the Bayesian estimate of the state being x at time t given the measurement in the past

$$\mathrm{d}\rho = \hat{L}\rho\mathrm{d}t + \rho h^{\mathrm{T}}\mathrm{d}y_t \tag{2.16}$$

where $\hat{L}$ is the Kolmogorov forward operator, which gives the evolution of the PDF of the state estimate without any measurement.

The Kushner equation solves the same problem, but with the normalized probability density $p(x,t) = \rho(x,t)/\int_X \rho(y,t)\mathrm{d}y$. We have

$$\mathrm{d}p = \hat{L}p\mathrm{d}t + p(h - \mathbb{E}_t[h])^{\mathrm{T}}(\mathrm{d}y_t - \mathbb{E}_t[h]\mathrm{d}t) \tag{2.17}$$

where the expected measurement $\mathbb{E}_t[h] = \int_X h(x)p(x,t)\mathrm{d}x$.

The information filtering is usually difficult to solve because the unknown PDF belongs to an infinite-dimensional function space. But there are two cases in which the problem can be reduced to finite dimension. The first case is when the support of the function is a finite set. The second case is when the drift and the measurement are linear in the state, i.e. $f(x) = ax$, $h(x) = cx$, the diffusion is constant, i.e. $\sigma(x) = \sigma$, and the prior $p(x, 0)$ is Gaussian. In this case, $p(x, t)$ is always a Gaussian, and its mean μ_t and its co-variance Σ_t are given by the Kalman-Bucy filter

$$\begin{cases} \mathrm{d}\mu_t = a\mu_t \mathrm{d}t + \Sigma_t c^{\mathrm{T}}(dy_t - c\mu_t \mathrm{d}t) \\ d\Sigma_t = (\sigma^{\mathrm{T}}\sigma + a\Sigma_t + \Sigma_t a^{\mathrm{T}} - \Sigma_t c^{\mathrm{T}} c\Sigma_t)\mathrm{d}t. \end{cases} \tag{2.18}$$

Note that Σ_t is given by a differential Riccati equation, similar to the solution of the LQR problem. Based on this observation, Kalman suggests there is a general duality between the control problem and the state estimation problem. This idea is proofed in a modified version in [19].

2.1.4 Differential Game

Game theory studies the behaviors of two or more players interacting according to a set of rules, like in a "game". Each player in the game tries to maximize his gain or minimizing his loss by choosing a strategy based on the information available to it. A differential game is a game whose rules are the dynamics of a system, so the study of differential games lies in the intersection of the optimal control theory and the game theory.

A few situations in a game are of great interest. A Nash equilibrium is the situation when no player can benefit from changing his strategy unilaterally. Nash equilibrium does not necessarily exist if all the players choose their strategy in a deterministic way, which is called a pure strategy, contrary to a mixed strategy in which the players choose their strategy with a certain probability. A Pareto optimum is a situation when no player can benefits from any change which does not harm any other player. It is optimal under a certain compromise.

The prisoner's dilemma [14] illustrates the difference between the Nash equilibrium and the Pareto optimum. When both prisoners betray the other, they reach the Nash equilibrium. When both prisoners remain silent, they reach the Pareto optimum. We can see from this example that the pessimistic rationality prevents the players from reaching an optimistic result, and results in a dilemmatic Nash equilibrium. The conditions under which a Nash equilibrium is also a Pareto optimum have been actively studied.

To play a game, information is critical. A player has so-called complete information if he knows the rules and the cost function of the other players. A player has so-called perfect information if he has access to all the variables in this game. If a player has complete information, he can calculate the Nash equilibrium of the game, if it exists, by finding the optimal control policy of each player by simultaneously optimizing their cost function. In addition, if he has perfect information, he can execute his optimal control policy, which usually requires the knowledge of the state of the game.

2.2 MOTOR SYSTEM

2.2.1 Motor Unit

Our skeleton muscles are composed of elastic muscle fibers, who drive the bones they are attached to when they contract under the command from the central nervous system (CNS).

The neural fibers which relay the command from the CNS to the muscle are called muscle efferent. The neurons in theses efferents are called α motor neurons. An α neuron and the muscle fibers it innervates constitutes a motor unit. It's the basic unit responsible for all skeleton movements.

Muscle afferents are the neural fibers which provide information about the physical state of muscle to the central nervous system. There are three types of afferents responsible for three types of information.

- Primary afferent (Ia) fibers innervate the central region of the muscle spindle. They encode approximately the velocity of lengthening of muscle fiber.
- Secondary afferent (II) fibers innervate the poles of the muscle spindle. They encode approximately the length of muscle fiber.
- Golgi tendon afferent (Ib) fibers innervate Golgi tendon organs. They encode the tension in the muscle

Muscle spindles are also innervated by γ motor neurons. These neurons co-activate with α neurons and result in a force in the muscle spindle keeping it taut. This mechanism also allows Ia fibers to detect any discrepancy between the intended movement and the actual movement.

[18] gives a nice presentation of the Hill's muscle model. In this model, the muscle is composed of a damped contractile element, an elastic parallel element, and an elastic serial element, illustrated in Figure 1.

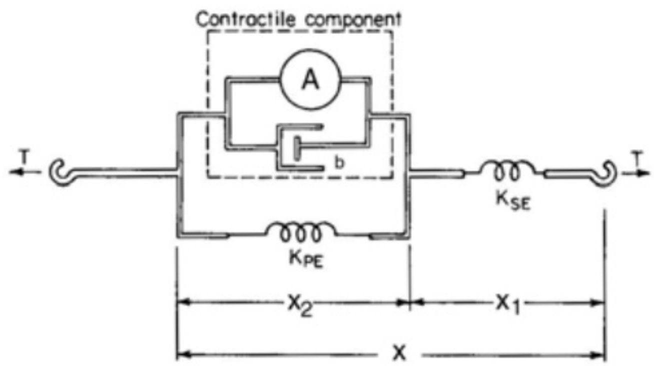


Figure 1: Hill's muscle model, taken from McMahon [10].

And the muscle dynamics is given by

$$\dot{T} = \frac{K_{\mathrm{SE}}}{b}\left(K_{\mathrm{PE}}\,\Delta\,x + b\dot{x} - \left(1 + \frac{K_{\mathrm{PE}}}{K_{\mathrm{SE}}}\right)T + A\right) \tag{2.19}$$

where T is the muscle tension, x is the muscle length, $\Delta\,x$ is the relative change in muscle length, A is the active force, K is the stiffness coefficient, b is the friction coefficient, PE stands for parallel element, SE stands for parallel element.

A relation between the muscle tension f and the rotational torque τ it generates on the articulation is also given in [18]

$$\tau = -J^{\mathrm{T}}f \tag{2.20}$$

where J is the Jacobian matrix of the geometric relationship between the muscles and the bones.

2.2.2 Motor Control

Motor control is the process in which the CNS commands voluntary movements. It includes sensory integration, motor planning, motor learning etc. It is also crucial in understanding human cognition.

But contrary to the musculoskeletal system whose dynamics can be relatively easily explained by the laws of physics, the mechanism of the CNS is fairly complex and not fully understood. Furthermore, the motor control itself is not a tractable problem. It's very complicated to find the appropriate commands for our muscle given a certain motor task. There are multiple challenges the CNS must tackle, such as the complex muscle dynamics, the perturbation from the environment, the biological noise and delay etc. In consequence, the best models we have of motor control are only based on experimental observations and intuitive principles.

The recent developments in motor control theories include: [3] proposed the observer-predictor-feedback (OPF) model of human motor control; [11] used the OPF model to study human interaction with machine; [16] mapped the functions in the OPF model with brain structures; [20] further developed the optimal feedback theory for motor control using stochasticity arguments. The main assumption of the optimal feedback theory is that the human motor commands always intend to optimize certain cost function reflecting the nature of our skeletomuscular system and the motor task.

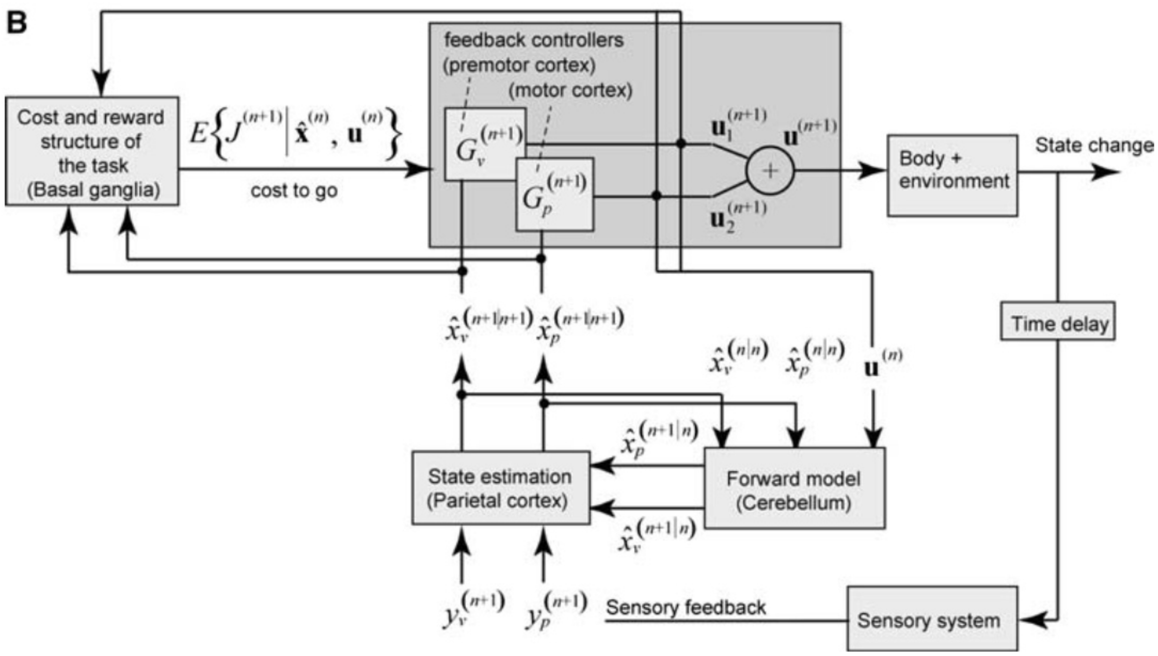


Figure 2: OPF model and corresponding brain structures, taken from Shadmehr [16].

3.0 A MODEL OF HUMAN MOTOR CONTROL

To study the human interaction with machines, we first need to fully understand the human motor system.

3.1 A MODEL OF MUSCULOSKELETAL SYSTEM

3.1.1 Model

One of the extremely important and relatively simple musculoskeletal systems in the human body is the forearm. It is crucial in reaching and pointing movements and supports the movement of the wrist and the digits, which makes it indispensable in the usage of most of the tools. So the forearm will be carefully studied in this section as a typical example of our motor system.

For the sake of simplicity, the following assumptions are made.

- The position of the should joint remains still and serves as the origin. The position of the wrist joint is referred as the "hand position" y.
- The forearm is treated as a two-link manipulator, where the ulna and radius are treated as a single rigid body. Articulations and ligaments are considered ideal.
- All the movements of the forearm are considered as a combination of flexion/extension, horizontal abduction/adduction and internal/external rotations of the should joint and flexion/extension of the elbow joint. The degrees of these movements are respectively denoted by θ_1, θ_2, θ_3 and θ_4, as shown in Figure 3. In particular, pronation/supination are not considered.

- Muscles in the forearm are lumped into four "effective muscles" each responsible for one of the four movements. Each of them has an effective lengthening x_i. Muscle attachments are considered as isolated points on the bones. Motor signals (firing rate of each motor unit in every muscle) are lumped into one single contractile force A_i for each of the above movements.

These assumptions are more or less far from the reality, but they can still provide us a good understanding of the essentials of the forearm.

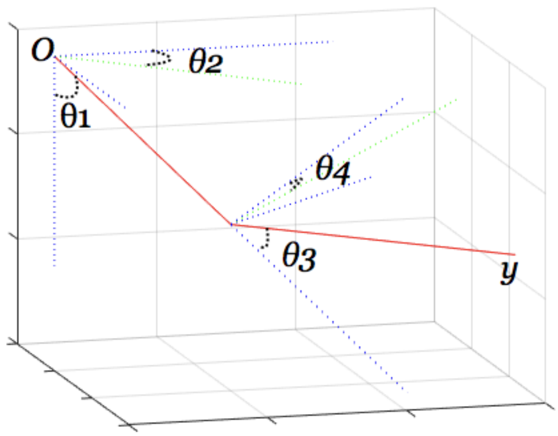


Figure 3: Idealized forearm movements. Two red line segments, described respectively in two spherical coordinates, represent the forearm.

Using the elements derived in [18] and [12], the dynamics of the forearm can be summarized in the block diagram Figure 4, where the skeleton dynamics is given by

$$\sum_j M_{ij}(\theta)\ddot{\theta}_j + \sum_{jk} \Gamma_{ijk}(\theta)\dot{\theta}_j\dot{\theta}_k + \frac{\partial V}{\partial \theta_i}(\theta) = \tau_i(\theta), \tag{3.1}$$

the muscle dynamics is linear and given by

$$b_i\dot{T}_i + (K_{\mathrm{SE},i} + K_{\mathrm{PE},i})T_i = K_{\mathrm{SE},i}(K_{\mathrm{PE},i}x_i + b_i\dot{x}_i + A_i), \tag{3.2}$$

and the differentiable functions φ_1 and φ_2 depend only on geometrical parameters of the anatomy. In particular, φ_2 is not injective because there are multiple joint configurations leading to the same hand position, but φ_1 is a diffeomorphism, so there is a bijection between the muscle configuration $(x, \dot{x})$ and the joint configuration $(\theta, \dot{\theta})$, and the quantities measured by the muscle afferents $(x, \dot{x}, T)$ are can be used exactly as the states variables.

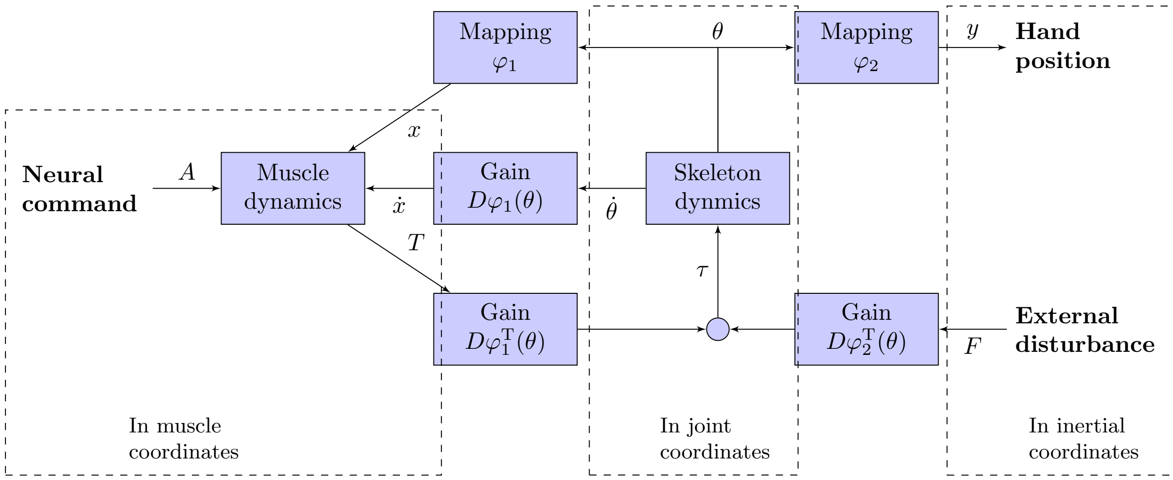


Figure 4: Dynamics of the forearm.

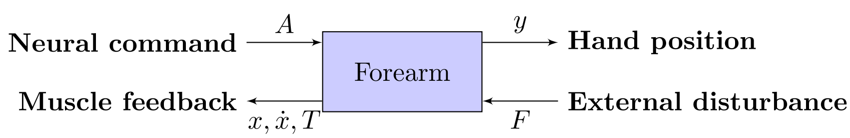


Figure 5: Black box representation of the forearm.

3.1.2 Simulation

This forearm model is simulated using Simulink® to study its controlled dynamics. A PID controller is synthesized to achieve step tracking and perturbation rejection. Although the classical control method has a satisfying performance in the simulated tasks, it doesn't truly reflect the biological control our CNS exercise over our body. There is indeed no difference between a biological arm and a robot arm if the intelligent neural commands are not taken into consideration. That's the motor control problem I will address in the next section.

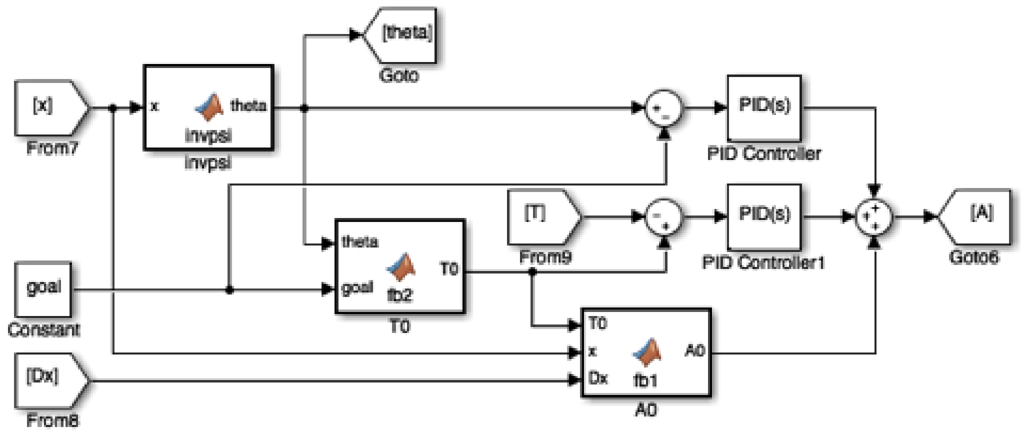


Figure 6: PID controller used to control the forearm.

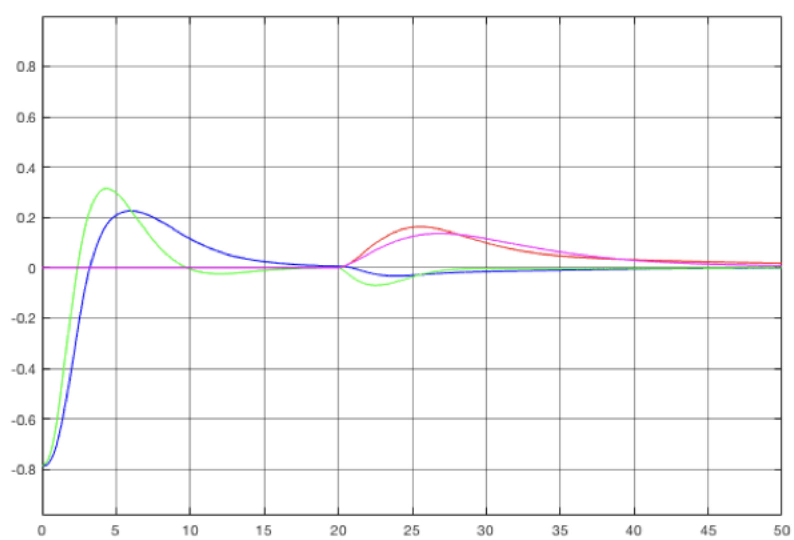


Figure 7: Forearm simulation result. Four colored curves represent the trajectories of the joint angles. A constant perturbation starts at $t = 20$.

3.2 A MODEL OF CENTRAL NERVOUS SYSTEM

Classical motor control models focus on how the CNS controls the body. The optimal feedback theory for motor control is a successful example. When a human interacts with a machine, the optimal feedback theory can be simply extended by postulating that the human, on top of optimizing his only motor cost function, also tries to optimize the performance of the machine. Then the motor control problem of the interaction can be formulated as

$$\begin{aligned} &\min_{u_1,x_1,x_2} \int_0^T r(u_1,x_1,x_2)\mathrm{d}t \\ &\text{s.t. } \dot{x}_1 = f_1(x_1,u_1,y_2) \\ &\qquad \dot{x}_2 = f_2(x_2,u_2,y_1) \end{aligned} \tag{3.3}$$

where x, u, y are respectively the state, the control and the input/output of the human and the machine as depicted in the following diagram Figure 8.

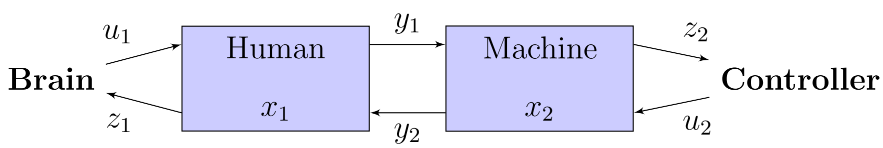


Figure 8: Interaction between a human and a machine.

Using the elements in the previous section, we can identify $u_1 = A$, $x_1 = (\theta, \dot{\theta}, T)$, $y_1 = y$ and $y_2 = F$. In addition to the full state feedback x_1 mentioned previously, our brain also has access to the vision, which provides y_1, and somatosensation, which provides y_2, in many motor tasks. We can assume that $z_1 = (x_1, y_1, y_2)$ in most cases.

The problem (3.3) is an optimal control problem with two coupled systems, in which one (the human body) gives full state feedback and another (the machine) gives output feedback. One way to solve this problem is to use pure gain feedback, i.e. find K_1 and K_2 such that $u = K_1x_1 - K_2y_2$ solves (3.3). The detailed solution for such problem is provided in [8].

But as [17] points out, our CNS is more than a simple feedback controller. It's capable of making predictions about the sensory consequences using an efferent copy and a forward model. When the measurement and the prediction are combined to make motor control, we achieve a better performance in the motor task. We'll see how this procedure can be modeled as a state estimation problem.

Another problem in this formulation is that the dynamics of the machine may be unknown to our CNS, even our own body has constantly changing dynamics. So our CNS cannot solve this problem without learning and adaptation. This can be modeled as a system identification problem.

3.2.1 State Estimation

Noise is everywhere in the real world. It represents the unknown fluctuation and variability. There is noise in the measurement of a quantity as well as in the evolution of a system, therefore neither the measurement or the prediction is hundred percent reliable, but both of them can contribute to our understanding of the system. The optimal way of combining them is the Bayesian inference.

If the CNS makes state estimation using Bayesian inference for the human body as suggests [17], we can make the assumption that the CNS represents all of its motor knowledge using Bayesian probability, which can be interpreted as our "belief" regarding the state of our body as well as the exterior world. This assumption is also supported by the fact that even external artificial actuator may have its neural representation if it's wired to the CNS [7], so there should be no fundamental distinction between the human body and the external devices from the perspective of the CNS. This section will elaborate this model and its control theoretical consequence.

Combine the human dynamics and the machine dynamics in the problem (3.3), and consider its stochastic version

$$\begin{cases} \mathrm{d}x_t = f(x_t, u_t)\mathrm{d}t + \sigma(x_t, u_t)\mathrm{d}\omega_t \\ \mathrm{d}y_t = h(x_t, u_t)\mathrm{d}t + \mathrm{d}\nu_t \end{cases} \tag{3.4}$$

where ν_t and ω_t are two independent Wiener processes. It's important to incorporate multiplicative noise in motor control as [20] demonstrates. For a state estimation problem, the control u_t doesn't play an important role, as it's known to the controller. It can be simply thought as a part of the non-autonomous dynamics.

The Zakai equation gives us the non-normalized state estimate PDF

$$\mathrm{d}p = \hat{L}p\mathrm{d}t + ph^{\mathrm{T}}\mathrm{d}y_t \tag{3.5}$$

where $\hat{L}$ is the Kolmogorov forward operator. Using the techniques presented in [5] and [19], (3.5) can be transformed into a PDE

$$\partial_t p = \hat{L}p + \left(h^{\mathrm{T}}\dot{y}_t - \frac{1}{2}h^{\mathrm{T}}h\right)p. \tag{3.6}$$

This equation, with a term of prediction and a term of measurement, represents how the Bayesian state estimate evolves in our CNS. It should be considered as the dynamics of the CNS.

3.2.2 Optimal Feedback

Now we can consider the optimal control problem (3.3). Again, write the problem in a concise and stochastic version

$$\begin{aligned} &\min_{u_t,x_t}\mathbb{E}\left[\int_0^T r(x_t,u_t,t)\mathrm{d}t\right] \\ &\text{s.t. } \mathrm{d}x_t = f(x_t,u_t)\mathrm{d}t + \sigma(x_t,u_t)\mathrm{d}\omega_t \\ &\qquad \mathrm{d}y_t = h(x_t,u_t)\mathrm{d}t + \mathrm{d}\nu_t. \end{aligned} \tag{3.7}$$

As the CNS doesn't have access to the state value, the expectation operator has to be defined with respect to the probability density of the state estimation, so we have

$$\mathbb{E}[r(x,u,t)] = \frac{\int_X r(x,u,t)p(x,t)\mathrm{d}x}{\int_X p(x,t)\mathrm{d}x} \tag{3.8}$$

which means that the CNS optimizes its estimate of the motor cost.

Using (3.8) and (3.6), the stochastic non-linear problem (3.7) can be transformed into a deterministic linear problem

$$\begin{aligned} &\min_{u,p}\int_0^T L(p,u,t)\mathrm{d}t \\ &\text{s.t. } \partial_t p = \hat{H}p \end{aligned} \tag{3.9}$$

where $L(p,u,t) = \mathbb{E}[r(x,u,t)]$ is the Lagrangian of the new problem, and $\hat{H} = \hat{L} + h^{\mathrm{T}}\dot{y} - \frac{1}{2}h^{\mathrm{T}}h$ is the linear time-evolution operator of the new dynamics, depending on the measurement $\dot{y}$ and the control u. Note that the price to pay for transforming a nonlinear dynamics into a linear one is the infinite dimensionality of the new state space, a function space. Also note that, the belief p is adapted to the natural filtration of y_t, which can be understood as given the measurement, the belief evolves in a deterministic way, so we always "see" a deterministic world.

(3.9) is an optimization problem with an elliptic PDE constraint. Classical optimization results, such as KKT or HJB, can be extended into Banach space, despite technical subtleties and numerical complexity. Another difficulty of this problem is that the Lagrangian density is non-local due to the normalization factor. This can be transformed into an integral constraint and be solved using related techniques. Using the Kushner equation also gets rid of the non-local Lagrangian, but results in more complicated dynamics involving the expectation operator, which is an integro-partial differential equation constraint, somehow as difficult as the problem using the Zakai equation.

3.2.3 System Identification

If the human subject is inexperienced in the motor task, he needs to learn the machine dynamics and constructs a forward model in his CNS so that he can make the motor prediction as correct as possible. Even if the human is experienced, he still needs to adapt his motor control according to the potential evolution of the machine dynamics and his own body dynamics.

The learning and adaptation processes can be modeled as system identification problem in control theory, as suggests [17]. However, the system identification problem in this model should be understood differently from the usual stochastic identification. In a classical stochastic system, the diffusion is due to the physical noise, whether it's thermodynamic, biological or statistical. But with the Bayesian interpretation, the diffusion term represents the uncertainty subjectively. The objective of system identification is to reduce this uncertainty and improve the accuracy, within the cognitive capacity.

To this end, we need to find the functions f, σ, and h, or the operator L, or the functional measure P induced by the stochastic process x_t, which maximizes the likelihood of obtaining the measured quantity $\dot{y}$

$$\max_P \int \exp\left(-\frac{1}{2}\int_0^T (\dot{y}(t) - h(x_t))^2 \mathrm{d}t\right) \mathrm{d}P(x_t). \tag{3.10}$$

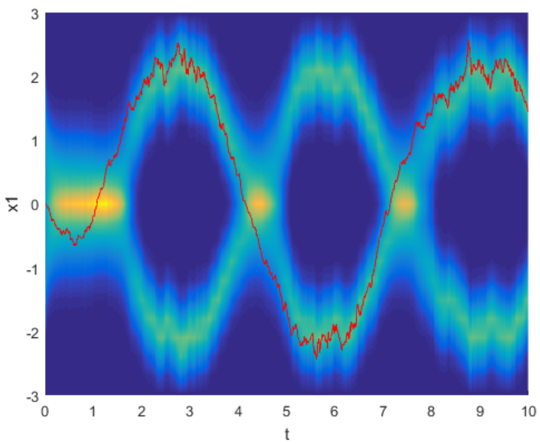
Note that the quantity to be maximized is a path integral, and the Radon-Nikodym derivative of P with respect to the Wiener measure gives the law of the process x_t. Without any constraints, the optimal x_t is nothing but $\dot{y}(t)$, To obtain meaningful solutions, appropriate constraints, such as the initial condition, the time-invariance, the smoothness, the norm of the diffusion, or even a parametric model, need to be considered. If there are multiple trials, then a statistical inference can be achieved by maximizing the joint probability of (3.10), and the aforementioned constraints may be weakened. This justifies the role of "experience" in motor learning.

3.2.4 Simulation

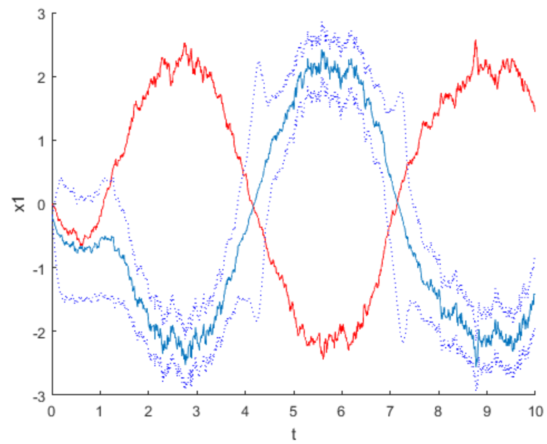
State estimation problems are simulated using MATLAB® to illustrate the benefits of the Bayesian framework.

In most cases, Gaussian distribution is a nice approximation for many well-studied PDFs, which justifies the usage of the (extended) Kalman filter even as nonlinear state estimator. However, in practice, our beliefs formed by limited knowledge may have more features than the Gaussian do. And we shall see that the Zakai filter has advantages over the extended Kalman filter in the following simulation.

Consider a dynamics with a quadratic output. As the output is non-injective, there is no way for the controller to know the sign of the state. Such problem is also encountered in some motor tasks, for instance, when we see an image without knowing whether it's a mirror image. As shows Figure 9, the Zakai filter truthfully reflects this knowledge constraint by splitting the belief into two halves, thus allows the controller the make a reasonable estimate of the cost and make control policy. On the other hand, the Kalman filter "guesses" a trajectory and follows it. Although it has half a chance of guessing correctly, it may be completely wrong for a single trial and leads to control errors.



(a) The result of the Zakai filter shown as the color map. The true trajectory is shown in red.



(b) The result of the Kalman filter, where the state trajectory is red and the estimate trajectory is blue.

Figure 9: Simulation comparison between Zakai filter and Kalman filter of a certain trial.

4.0 HUMAN INTERACTION WITH MACHINES

The regular tools we use, such as hammers or pens, have a fixed and passive dynamics. With the model of human motor intelligence proposed in the previous chapter, our interaction with them can be fairly well understood. But the future of toolmaking is to create "intelligent" machines which have their own ability to control and adapt. This is reflected by the "Controller" in Figure (8). As a result, the learning processes of the human and the machine are intertwined, and we this called co-learning.

We want the machine to have the same intelligence as the human, using the model proposed in the previous chapter. With this strategy, the machine tries to imitate human's motor intelligence. The overall result should hopefully not be different from the interaction between two humans.

To discuss the human-machine interaction from a game-theoretical point of view, we do not need all the details in Figure (8). We should interpret the interaction as a differential game in which the human brain and the machine controller control the overall dynamics of the body and the machine together, as shown in the following diagram Figure (10).

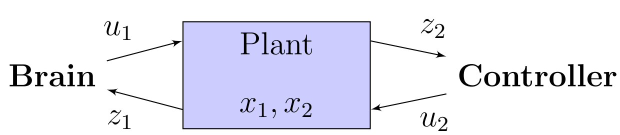


Figure 10: A game-theoretical model of human-machine interaction.

This game is very difficult because both the human and the machine have incomplete and imperfect information. If we consider that the system identification can be achieved effectively, then we only have to estimate the cost function of the other player to reach complete information. Having the complete information, the players can find their optimal strategy and execute it using state estimates.

Cost function estimation is called the inverse optimal control problem, a solved example of which is the inverse LQR problem [15]. Another way to resolve the incomplete information is the Bayesian game. In a Bayesian game, each player assigns a probability to the possible cost functions of the other players. During the game, the players can update their assumption and adjust their control policy accordingly.

Let's analyze the optimal control policies resulting in the Nash equilibrium. The Nash equilibrium of a two-player game satisfies

$$\begin{cases} S_1[u_1^*, u_2^*] \leq S_1[u_1, u_2^*] \\ S_2[u_1^*, u_2^*] \leq S_2[u_1^*, u_2]. \end{cases} \tag{4.1}$$

In the deterministic case, It can be found by solving a bi-level programming problem

$$\begin{aligned} &\min_{u_1} S_1[u_1, u_2^*] \\ \text{s.t. } \; &u_2^* = \arg\min_{u_2} S_2[u_1, u_2] \\ &\qquad \text{s.t. } \; \dot{x} = f(x, u_1, u_2, t). \end{aligned} \tag{4.2}$$

In the stochastic formulation, the dynamics of the players are actually two independent PDEs. And the cost function of one player needs to include not only his belief, but also the belief of the another player as well, in order to correctly achieve the relationship between two players, as defined in [4]. The corresponding bi-level programming problem is

$$\begin{aligned} &\min_{u_1} S_1[u_1, u_2^*] \\ \text{s.t. } \; &u_2^* = \arg\min_{u_2} S_2[u_1, u_2] \\ &\qquad \text{s.t. } \; \begin{cases} \partial_t p_2 = \hat{L} p_2 + h_2^{\mathrm{T}} (\dot{y}_2 - h_2/2) p_2 \\ \partial_t p_1 = \hat{L} p_1 + h_1^{\mathrm{T}} (\dot{y}_1 - h_1/2) p_1 \end{cases} \end{aligned} \tag{4.3}$$

where $\hat{L}$, h_1 and h_2 depend on u_1 and u_2. Furthermore, if the two players have different estimates of the system, i.e. the operator $\hat{L}$ they use in their respective bi-level programming are different, then both of them will obtain a sub-optimal solution.

5.0 CONCLUSION AND FURTHER WORK

A complete model for human interaction with machines is proposed, including a skeletomuscular model based on mechanical principles, a model of motor control based on Bayesian probability, and a model of interaction based on game theory.

We've seen how a stochastic nonlinear problem can be transformed into a deterministic linear problem. This allows us to consider nonlinear dynamics and nonlinear measurements which are omnipresent in biological and physical systems. The interpretation of the stochasticity in the system dynamics is extended from physical noise to subjective uncertainty in our brain. And our probabilistic beliefs about the body and the exterior world are no longer limited by Gaussian distribution, they can take more elaborate forms. If we design machines using the same estimation and control algorithms, then the interaction between humans and machines is the evolution of two probabilistic beliefs coupled through observations. This process can be called co-learning between humans and machine.

However, the probabilistic formulation makes the numerical solution of the problem very difficult. Am I over-complicating the motor control problem? I believe the answer is no. First, the old models have a lot of limitations. If we want new advancement in discovering human motor control, we need consider more advanced models. Second, our brain has an amazing computational ability. Despite all the mysteries of the brain, we achieved a lot of impressive work by simply mimicking the computation of basic neural networks. More advanced motor control model, if proven to be valid, may also help us learn more about how our brain computes. A last, advanced stochastic tools, such as path integral formulation and stochastic control of partially observable systems are well-studied subjects. And they are no longer unknown to neuroscientists [9] [2]. People are ready for the challenges and looking forward to them.

A lot of work can be done to further the discussion in this thesis. From the theoretical point of view, this thesis has many arguments and results which are not mathematically rigorous. In addition, analytic properties of the equations and the solutions should be carefully studied in order to obtain more interesting results. The studies of the interaction between humans and machines can be deepened, because the different scenarios and applications of such interaction are only limited by our imagination. This can also be generalized into the interactions between machines and machines, humans and humans etc. From the practical point of view, the model proposed needed to be tested experimentally to verify its accuracy and limitation. This requires us to design experiments which can distinguish old models and the new model. The problem illustrated in the simulation can be a good example. To which extent our brain can it hold very different beliefs? Other questions, such as how we act when very little information or contrary information is available, can be asked. The numerical solution of the optimization problem is another difficulty. The classical finite element method can be extremely costly Good algorithms, which involve probably artificial neural network, should be developed.

BIBLIOGRAPHY

[1] C.-T. Chen. *Linear System Theory and Design*. Oxford University Press, 4th edition, 2013.

[2] C. C. Chow and M. A. Buice. Path integral methods for stochastic differential equations. *The Journal of Mathematical Neuroscience*, 2015.

[3] S. Baron D. L. Kleinman and W. H. Levison. An optimal control model of human response part i: Theory and validation. *Automatica*, 1970.

[4] N. Jarrasse, T. Charalambous, and E. Burdet. A framework to describe, analyze and generateinteractive motor behaviors. *PLoS One*, 2012.

[5] V. Krishnamurthy. Robust continuous-time smoothers without two-sided stochastic integrals. *IEEE Transactions on Automatic Control*, 47(11), 2002.

[6] M. L. Latash. Elements of history. In *Fundamentals of Motor Control*, chapter 2. Elsevier Science, 2012.

[7] M. A. Lebedev, J. M. Carmena, J. E. O'Doherty, M. Zacksenhouse, C. S. Henriquez, J. C. Principe, and M. A. L. Nicolelis. Cortical ensemble adaptation to represent velocity of an artificial actuator controlled by a brain-machine interface. *JNeurosci*, 2005.

[8] F. L. Lewis, D. Vrabie, and V. L. Syrmos. *Optimal Control*. Wiley, 3rd edition, 2012.

[9] D. Marti. A short tutorial on the path integral formalism for stochastic differential equations. *Janelia Neurotheory Workshop*, 2014.

[10] T. A. McMahon. *Muscles, Reflexes, and Locomotion*. Princeton University Press, 1984.

[11] D. McRuer. Human dynamics in man-machine systems. *Automatica*, 1980.

[12] R. M. Murray, Z. X. Li, and S. S. Sastry. Robot dynamics and control. In *A Mathematical Introduction to Robotic Manipulation*, chapter 4. CRC Press, 1994.

[13] K. P. Oakley. *Man the Tool-maker*. The Trustee's of the British Museum, 1949.

[14] W. Poundstone. *Prisoner's Dilemma*. Anchor Books, 1992.

[15] M. C. Priess, R. Conway, J. Choi, J. M. Popovich Jr., and C. Radcliffe. Solutions to the inverse lqr problem with application to biological systems analysis. *IEEE Transactions on Control Systems Technology*, 2014.

[16] R. Shadmehr and J. W. Krakauer. A computational neuroanatomy for motor control. *Experimental Brain Research*, 2008.

[17] R. Shadmehr and S. Mussa-Ivaldi. *Biological Learning and Control*. MIT Press, 2012.

[18] R. Shadmehr and S. P. Wise. What generates force and feedback. In *The Computational Neurobiology of Reaching and Pointing*, chapter 7. MIT Press, 2005.

[19] E. Todorov. General duality between optimal control and estimation. *Proceedings of the 47th IEEE Conference on Decision and Control*, 2008.

[20] E. Todorov and M. I. Jordan. Optimal feedback control as a theory of motor coordination. *Nature Neuroscience*, 2002.

[21] C. Winick. *Dictionary of Anthropology*. Philosophical Library, Inc, 1956.

[22] R. S. Woodworth. *The Accuracy of Voluntary Movement*. PhD thesis, Columbia University, 1899.